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CHARACTERIZATION OF THE PARAMETER-MISMATCHING AND NOISE EFFECT ON WEAK SYNCHRONIZATION

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We investigate the effect of noise and parameter mismatching on the loss of chaos synchronization in coupled one-dimensional maps. Due to existence of positive local transverse Lyapunov exponents, the weakly stable synchronous chaotic attractor demonstrates sensitivity with respect to variation of the mismatching parameter or noise intensity. In order to characterize such parameter and noise sensitivity quantitatively, we introduce new quantifiers, called the parameter sensitivity exponent and noise sensitivity exponent. The values of these exponents are determined by local stability multipliers of the chaotic trajectories, and by the properties of the noise signal (for the noise sensitivity exponent). For the case of bounded uniform noise, the values of the parameter sensitivity exponent and noise sensitivity exponent coincide. In terms of these exponents, we characterize the effect of parameter-mismatching and noise on the intermittent bursting and basin riddling occurring in the regime of weak synchronization.

Introduction

In recent years, the phenomenon of chaos synchronization in coupled dynamical systems attracts attention of the researches. Synchronous chaotic attractor (SCA) exists on the invariant subspace [1]. If the SCA is stable against perturbations transverse to the invariant subspace, it may become an attractor in the whole space. Such a transverse stability is closely associated with properties of unstable periodic orbits, embedded into SCA [2,3]. If all the orbits are transversally stable, then the SCA becomes asymptotically stable, and strong synchronization occurs. However, as the coupling parameter passes through a threshold value, periodic orbit first becomes transversally unstable through a local bifurcation, and trajectories in vicinity of such orbit may be locally repelled from the invariant subspace (diagonal). Thus, weak synchronization arises; for this case, transient intermittent bursting or basin riddling may occur depending on existence of the absorbing area, controlling the global dynamics, inside the basin of attraction.

In a real situation, some small noise and parameter mismatch between subsystems exist, which destroy the invariant diagonal. A typical trajectory leaves the diagonal and undergoes transversal repulsion in vicinity of periodic repellers, embedded into the SCA; as a result, the typical trajectory may have segments exhibiting positive local (finite-time) transverse Lyapunov exponents. Thus, for the case of bubbling, permanent intermittent bursting occurs; for the riddling case, trajectory goes to another attractor or infinity. The both bubbling and riddling regimes of weak synchronization demonstrate sensitivity with respect to variation of the noise intensity and mismatching parameter. In order to measure the «degree» of such sensitivity quantitatively, we introduce new quantifiers, called the parameter sensitivity exponent (PSE) [4] and noise sensitivity exponent (NSE) [5]. In terms of PSE and NSE, we characterize the effect of noise and parameter mismatching on intermittent bursting and basin riddling. We show that the average escape time from the diagonal (interburst interval for bubbling regime and average lifetime of the chaotic transient for riddling regime) can be characterized quantitatively using the PSE and NSE. We also show, that the bounded noise and parameter mismatch have essentially the same effect on the power-law scaling behavior of the escape time, although their properties such as probability distribution and temporal correlation are in general different.

Characterization of the parameter sensitivity of the synchronous chaotic attractor

We investigate the parameter-mismatching effect on the weak synchronization in two coupled 1D maps:

$$T: \begin{cases} x_{n+1} = F(x_n,y_n) = f(x_n,a) + (1\text{-}\alpha)cg(x_n,y_n), \\ y_{n+1} = G(x_n,y_n) = f(y_n,b) + cg(y_n,x_n). \end{cases} \tag{1}$$

Local dynamics in each subsystem with a control parameter $p(p=a,b)$ is governed by the 1D map $f(x, p)=1\text{-}px^2$, c is a coupling parameter, and $g(x,y)=y^2\text{-}x^2$ is a coupling function. The parameter α tunes the asymmetry of coupling.

For the case of identical 1D maps (i.e., $a=b$), there exists an invariant synchronization line, $y=x$, in the (x,y) phase space. However, in presence of a mismatching between two 1D maps, the diagonal is no longer invariant. To take into consideration such a mismatching effect, we introduce a small mismatching parameter ε such that $b=a\text{-}\varepsilon$, and consider an orbit $\{(x_n,y_n)\}$ starting from an initial point on the diagonal (i.e., $x_0=y_0$). Due to local transverse repulsion, the weakly stable SCA is sensitive to the variation of the mismatching ε. In order to characterize such sensitivity of the SCA at $\varepsilon=0$, we calculate the derivative of the transverse variable $u_n[=x_n\text{-}y_n]$, denoting deviation from the diagonal, with respect to ε. Using Eq. (1), we may obtain a recurrence relation

$$\partial u_{n+1}/\partial\varepsilon|_{\varepsilon=0} = [f_x(x_n^*,a) - (2\text{-}\alpha)ch(x_n^*)]\partial u_n/\partial\varepsilon|_{\varepsilon=0} + f_a(x_n^*,a), \tag{2}$$

where f_x and f_a are the derivatives with respect to x and a, $\{(x_n^*, y_n^*)\}$ is the synchronous orbit with $x_n^*=y_n^*$ for $\varepsilon=0$, and $h(x)$ is a reduced coupling function defined by $h(x)\equiv\partial g(x,y)/\partial y|_{y=x}$. Hence, starting from an initial point (x_0^*,y_0^*) on the diagonal, we may to obtain the derivatives at all points of the orbit as

$$\partial u_N/\partial\varepsilon|_{\varepsilon=0} = S_N(x_0^*) \equiv \Sigma_{k=1}^{N} R_{N\text{-}k}(x_k^*)f_a(x_{k\text{-}1}^*,a), \tag{3}$$

since $\partial u_0/\partial\varepsilon=0$, and where

$$R_M(x_m^*) = \Pi_{i=0}^{M\text{-}1} [f_x(x_{m+i}^*,a) - (2\text{-}\alpha)ch(x_{m+i}^*)]. \tag{4}$$

One can easily see that the factor $R_M(x_m^*)$ is associated with a local (M-time) transverse Lyapunov exponent $\sigma_M^T(x_m^*)$ of the SCA as $\sigma_M^T(x_m^*)=(1/M)\ln|R_M(x_m^*)|$. Thus, $R_M(x_m^*)$ becomes a local (stability) multiplier. When a typical trajectory visits neighborhoods of repellers embedded into SCA, it has segments experiencing local repulsion from the diagonal. Thus, the distribution of local transverse Lyapunov exponents σ_M^T for a large ensemble of trajectories and large M may have a positive tail. For the segments of trajectories exhibiting positive local Lyapunov exponents ($\sigma_M^T>0$), the local multipliers $R_M[=\pm\exp(\sigma_M^T M)]$ can be arbitrarily large, and hence the partial sums S_N may be arbitrarily large. This implies unbounded growth of the derivatives $\partial u_N/\partial\varepsilon|_{\varepsilon=0}$ (3) as N tends to infinity, and consequently the weakly stable SCA may have a parameter sensitivity.

As an example, we consider the SCA that exists in the interval of $c_{b,l}[\simeq-2.963]<$ $<c<c_{b,r}$ $[\simeq-0.677]$ for a=1.82 in the unidirectionally coupled case (α=1). When the coupling parameter c passes through $c_{b,l}$ or $c_{b,r}$, the SCA becomes transversally unstable through a blow-out bifurcation. A strongly stable SCA exists for $c_{t,l}$ $[\simeq-2.789]<c<$ $<c_{t,r}$ $[\simeq-0.850]$. For the case of strong synchronization, there is no parameter sensitivity, because all the periodic saddles embedded in the SCA are transversally stable. Hence, in presence of a small parameter mismatching ε the strongly stable SCA becomes only slightly perturbed. However, when the coupling parameter c passes through $c_{t,r}$ and $c_{t,l}$, bubbling and riddling transitions occur through the first transverse bifurcation of periodic saddles, respectively; for this case, the weakly stable SCA exhibits parameter sensitivity. However small mismatching ε, a persistent intermittent bursting, called the attractor bubbling, occurs in the regime of bubbling. On the other hand, in the riddling regime the weakly stable SCA with a riddled basin is transformed into a chaotic transient with a finite lifetime. To quantitatively characterize the parameter sensitivity of the SCA, we consider the behavior of the partial sums $S_N(x_0^*)$ of Eq. (3). The quantity S_N becomes very intermittent, as it is shown in Figure (a). However, looking only at the maximum $\gamma_N(x_0^*)=\max_{0\le n\le N}|S_n(x_0^*)|$, one can easily see the boundedness of S_N. Figure (b) shows the function γ_N for both cases of strong and weak synchronization. For the case of strong synchronization with c=-1.5, γ_N grows up to the largest possible value of $|\partial u/\partial\varepsilon|$ along the SCA and then saturates. Thus, the strongly stable SCA has no parameter sensitivity. On

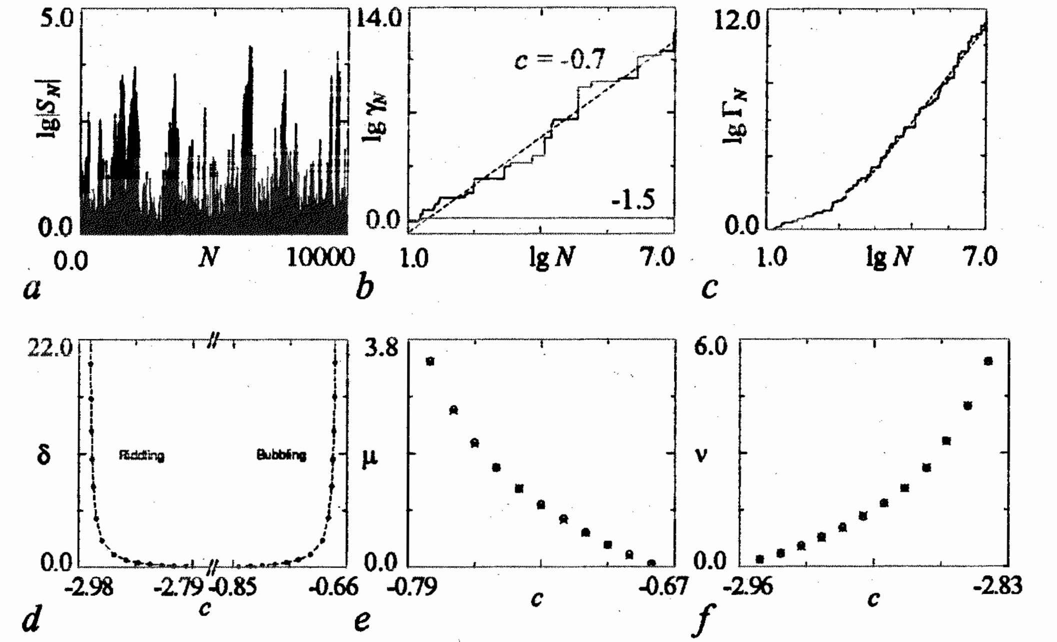

Figure. Parameters are fixed at α=1 and a=1.82. (a) Intermittent behavior of the partial sums for c= =-0.7. (b) Two functions γ_N for c=-1.5 (strong synchronization) and c=-0.7 (weak synchronization). (c) Parameter sensitivity function Γ_N at c=-0.7. (d) The plot of the PSEs δ versus c in the regime of weak synchronization. (e) The plot of the LPEs μ (circles); they agree well with the reciprocals of the PSEs (crosses). (f) The plot of the CTEs ν (circles); they agree well with the reciprocals of the PSEs (crosses) the other hand, for weak synchronization at c=-0.7, γ_N grows unboundedly and exhibits no saturation. Consequently, the weakly stable SCA has a parameter sensitivity.

The growth rate of the function $\gamma_N(x_0^*)$ with time N represents the degree of the parameter sensitivity, and can be used as a quantitative characteristic of the weakly stable SCA. However, $\gamma_N(x_0^*)$ depends upon the particular trajectory. To obtain a representative quantity, we consider an ensemble of randomly chosen initial points (x_0^*, y_0^*) on the diagonal, and take the minimum value of γ_N with respect to the initial orbit points,

$$\Gamma_N = \min_{x_0^*} \gamma_N(x_0^*). \tag{5}$$

Figure (c) shows the parameter sensitivity function Γ_N for c=-0.7. Note, that Γ_N grows unboundedly with some power: $\Gamma_N \sim N^{\delta}$. Here the value $\delta \simeq 2.58$ is a quantitative characteristic of the SCA, and we call it the PSE. In each regime of bubbling or riddling, we vary the coupling parameter c from the bubbling or riddling transition point to blow-out point, and obtain the PSE. For obtaining a satisfactory statistics, we consider 100 ensembles for each c, each of which contains 100 randomly chosen initial points, and choose the average of the 100 PSEs obtained in the 100 ensembles. Figure (d) shows the plot of the PSEs versus c. Note that the PSE δ monotonically increases as c is varied from bubbling or riddling transition point, and tends to infinity as c approaches the blow-out transition points. This increase is caused by the increase in the strength of local transverse repulsion of the periodic repellers, embedded into the SCA. After the blow-out bifurcation, the weakly stable SCA transforms into chaotic saddle, which has exponential parameter sensitivity.

We characterize the parameter-mismatching effect on the bubbling and riddling in terms of PSEs for a=1.82 and α=1. The quantity of interest in the both cases is the average time τ that a typical trajectory spends near the diagonal. For the case of bubbling, this is an average interburst interval. The trajectory is supposed to be in laminar phase if the magnitude of deviation from the diagonal is less then a threshold value u_b^* (i.e., $|u_n| < u_b^*$); otherwise, it is in a bursting phase. For each c, we obtain 50,000 laminar phases, and then get the average laminar length τ, which scales with ε as $\tau \sim \varepsilon^{-\mu}$ [6], where μ is referred to as the laminar phase exponent (LPE). The plot of the LPEs μ versus c is shown in Figure (e). On the other hand, the scaling relation for time τ can be obtained from the power-law growth of Γ_N as $\tau \sim \varepsilon^{-1/\delta}$. Hence, we obtain a reciprocal relation between the PSE and LPE: $\mu = 1/\delta$. The plot of the reciprocals of δ is also shown in Figure (e), and they agree well with the values of μ. This reciprocal relation is also valid in the riddling regime. For the riddling case, we consider the average lifetime of chaotic transient process, obtained over an ensemble of 1,000 initial conditions on the diagonal. This average lifetime τ_c scales with ε as $\tau_c \sim \varepsilon^{-\nu}$ [6], where ν is referred to as the chaotic transient exponent (CTE). Hence, the same reciprocal relation ($\nu = 1/\delta$) is valid for the case of the CTE. The plot of the CTEs ν versus c is shown in Figure (f). They coincide well with the reciprocals of the PSEs δ.

Characterization of the Noise Effect on the SCA and Discussion

We also investigated the effect of additive noise on weak synchronization in the same system of two coupled 1D maps:

$$\mathrm{T}: \begin{cases} x_{n+1} = F(x_n, y_n) = f(x_n, a) + (1-\alpha)cg(x_n, y_n) + \sigma\xi_n^{(1)}, \\ y_{n+1} = G(x_n, y_n) = f(y_n, a) + cg(y_n, x_n) + \sigma\xi_n^{(2)}, \end{cases} \tag{6}$$

where $\xi_n^{(1,2)}$ are independent uniformly distributed random variables with zero mean

$\langle\xi_n^{(1,2)}\rangle=0$ and unit variance $\langle(\xi_n^{(1,2)})^2\rangle=1$, and σ controls the «strength» of such a random noise. In order to characterize the noise sensitivity of the SCA, we calculate the derivative of the transverse variable u_n with respect to the noise intensity σ at σ=0: $\partial u_n/\partial\sigma|_{\sigma=0}$. Following the same arguments as in the case of parameter sensitivity, we introduce the noise sensitivity function $\Gamma_N^{(n)}$, which exhibits the power-law growth in the regimes of weak synchronization, and the noise sensitivity exponent (NSE) $\delta^{(n)}$, which measures quantitatively sensitivity of the SCA to variation of the noise intensity. For the case of bounded noise signal, the values of NSEs coincide with the same of PSEs, and hence, we can expect, that the bounded noise has the same effect on the power-law scaling of the escape time from the diagonal, as the parameter-mismatching effect. Indeed, the values of scaling exponents LPE and CTE for the both cases of the parameter mismatch and noise with bounded distribution are shown to be the same [5,6].

Finally note, that our method of characterization of the noise and parameter sensitivity of the SCA may be generalized to the coupled high-dimensional systems as Henon map or oscillators.

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ДИНАМИЧЕСКИЕ ХАРАКТЕРИСТИКИ РЕЖИМОВ СЛАБОЙ ХАОТИЧЕСКОЙ СИНХРОНИЗАЦИИ В ПРИСУТСТВИИ ШУМА И РАССТРОЙКИ ПАРАМЕТРОВ

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Исследуется влияние шума и расстройки параметров на динамические режимы при разрушении хаотической синхронизации в системе из двух связанных одномерных отображений. Благодаря наличию положительных значений локальных трансверсальных мультипликаторов хаотических траекторий, слабо устойчивый синхронный хаотический аттрактор демонстрирует чувствительность к вариации расстройки параметров и уровня шума. Для того чтобы количественно характеризовать параметрическую и шумовую чувствительность синхронного хаотического аттрактора, вводятся новые показатели, называемые показателями параметрической и шумовой чувствительности. Их значения определяются локальными трансверсальными мультипликаторами хаотических траекторий и свойствами шумового сигнала (для показателя шумовой чувствительности). В случае шума с ограниченным равномерным распределением значения показателей параметрической и шумовой чувствительности совпадают. Показано, что характеристики перемежающегося хаотического поведения и переходного процесса, индуцированных расстройкой параметров и шумом в режиме слабой синхронизации, могут быть выражены через показатели параметрической и шумовой чувствительности.

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