Application of Idea of Chaos Control to Stabilization of Stationary Generation in Backward-wave Oscillator

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Abstract

The method of suppression of self-modulation in a backward wave tube is offered. For this, the additional circuit of a delayed feedback is introduced in such way that the level of amplitude of output signal influences magnitude of a current of electron beam at the input of the interaction space. Results of numerical simulation demonstrate a possibility to increase working current twice, keeping a single-frequency regime of generation.

The approach to vacuum microwave electron-beam devices, as to nonlinear distributed dynamical systems, is based on the non-stationary nonlinear theory. It has been developed for several modifications of backward wave oscillators (BWO) [1,2,3], traveling wave tubes [4], gyrotrons [5]. In many cases, this approach reveals nontrivial bifurcations – loss of stability of single-frequency generation with birth of self-modulation, and transition to chaos [1, 6-12].

In BWO (fig. 1) the electron beam propagates with a velocity close to a phase velocity of electromagnetic wave (it ensures effective interaction), but the group velocity is opposite. The last ensures presence of interior feedback, absolute character of instability, and a possibility of self-oscillations as the beam current exceeds some starting value. Further increase of the current leads to appearance of periodic and then chaotic self-modulation [1].

The non-stationary processes and complex dynamical regimes in BWO may be of practical interest. In particular, BWO in chaotic regime may be used as generator of noise with spectrum concentrated in a frequency band shifted effectively by variation of accelerating voltage [7-10]. Features of the transient process in relativistic BWO (presence of high peak in amplitude at some moment of time) may be useful to obtain intense pulses of microwave radiation [11,12].

In many cases, however, self-modulation appears as a parasitic effect precluding a single-frequency generation with sufficiently high levels of power and efficiency, and it is desirable to exclude it.

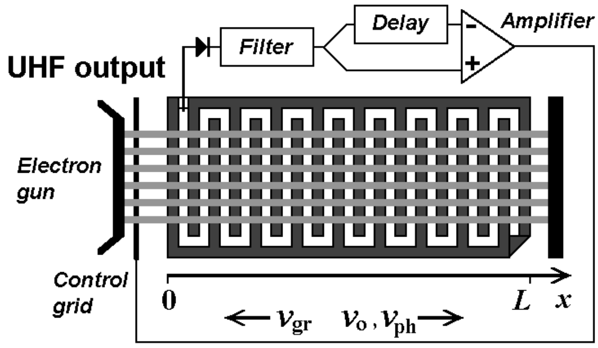

Figure 1: Schema of the backward wave oscillator (with added control circuit).

As we regard the BWO as a dynamical system, it is natural to turn to the idea of stabilization of unstable states known as chaos control. As known, this concept was suggested in 1990 as implementation of periodic dynamics instead of chaos by means of properly selected time-dependent small perturbations [13]. Latter, many other variants of the chaos control for stabilization of periodic states were offered. One simple and effective method exploits the delayed feedback [14].

Let us return to BWO and consider mechanism of the self-modulation (see the space-time diagram of Fig.2). Let us suppose that at some moment (I) the amplitude of the wave at the left-hand edge of the system (beam input) is maximal. Due to the over-bunching of electrons in the beam, the amplitude of the high-frequency current on the line $x - v_0 t = \mathrm{const}$ becomes less at the right-hand edge. Then, on a line of propagation of the wave packet with group velocity $x + v_{\mathrm{gr}} t = \mathrm{const}$, the amplitude of field also will be less. Therefore at $t \cong L/v_0 + L/v_{\mathrm{gr}}$ the amplitude of signal at the left-hand edge is minimal (III). The smaller field ensures more effective bunching in the beam, and maximal current is reached at the right edge (IV). It gives rise to a new maximum of the field amplitude (V) on the left-hand edge at $t = T_M \cong 2(L/v_0 + L/v_{\mathrm{gr}})$, and so on. The value T_M yields an estimate for a period of self-modulation. The numerical calculations improve the numerical factor: instead of 2 it appears to be close to 1.5.

Apparently, to prevent appearance of the self-modulation, it is necessary to vary input current of electron beam in time, in such way that it becomes larger at the moments of maximal output field amplitude and less at the minimal amplitudes. In Fig. 1 we show schematically how can it be done with a use of the delayed feedback scheme [14]. The high-frequency signal from output of the BWO is rectified and filtered to extract the envelope of the signal in a regime of actual or potential self-modulation. Further, the signal passes over two branches, in one of them it accepts delay by approximately a half of period of self-modulation, and then signals from both branches come to the input of a differential amplifier. Output signal of the amplifier acts as an additional voltage displacement on a control grid in the electronic gun, and regulates current at the beam input.

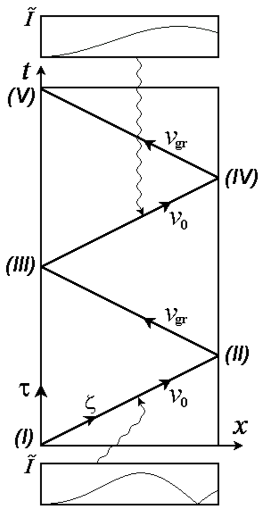

Figure 2: Illustration of the mechanism of the self-modulation on a space-time diagram

Let us demonstrate a possibility of suppression of self-modulation in a numerical experiment. To do this, we turn to equations of the non-stationary nonlinear theory of BWO [1,8]. It is convenient to use standard normalization of variables and parameters, with averaged current of the beam I_0 in the formulas. Variation of the current (due to presence of the control circuit) is taken into account in the right-hand part of the field equation, where a time-dependent factor is included. The equations look as follows:

$$\begin{gathered}\partial^2\theta/\partial\zeta^2 = -\operatorname{Re} F\exp(i\theta),\\ \partial F/\partial\tau - \partial F/\partial\zeta = A(\tau)I,\\ I = -\frac{1}{\pi}\int_0^{2\pi}\exp(-i\theta)d\theta_0,\end{gathered} \qquad (1)$$

$$\theta\big|_{\varsigma=0} = \theta_0,\quad \partial\theta/\partial\varsigma\big|_{\varsigma=0} = 0,\quad F\big|_{\varsigma=l} = 0\,. \qquad (2)$$

The dimensionless independent variables $\zeta=\beta_0 Cx$ and $\tau=\omega_0 C/(1+v_0/v_{gr})^{-1}(t-x/v_0)$ are defined in such way that the coordinate axis ζ is directed along the beam characteristic (Fig. 2). β_0 and ω_0 are wave number and circular frequency of the wave at synchronism with the beam, and $C=\sqrt[3]{I_0K/4U}$ is the Pierce parameter, expressed via the beam current I_0, impedance of coupling of the slow-wave structure K, and accelerating voltage U. $\theta(\varsigma,\tau,\theta_0)$ characterizes phase of an electron at coordinate ζ and at a moment τ that has entered the interaction space with an initial phase θ_0. $F(\varsigma,\tau) = E/2\beta_0 UC^2$ is the dimensionless complex amplitude of the electromagnetic field $E(x,t)=\operatorname{Re}\tilde{E}(x,t)\exp(i\omega_0 t - i\beta_0 x)$. It is worth noticing that a key condition of applicability of the theory is smallness of the Pierce parameter.

In lack of the control, the steady generation arises at dimensionless lengths larger than $l_{st}=1.97327$, and the self-modulation - at l larger than $l_{sm}\cong 2{,}937$ [1].

Let us assume that in presence of the control circuit the beam current is determined by relation

$$J(t) = I_0 + g\Delta V\,. \qquad (3)$$

Here $\Delta V = V(t) - V(t-\Delta t) \cong \beta_0^{-1}\left(\left|\tilde{E}(0,t)\right| - \left|\tilde{E}(0,t-\Delta t)\right|\right)$ is a voltage at the output of the control circuit, $V(t)$ is amplitude of high-frequency potential at the BWO output, Δt is time of delay, g is a fixed coefficient of dimension of conductance. Setting $A=J/I_0$ in dimensionless variables we write

$$\begin{aligned}A(\tau) &= 1 + 2gUI_0^{-1}C^2\left(\left|F(0,\tau)\right| - \left|F(0,\tau-T)\right|\right)\\ &= 1 + cl^{-1}\left(\left|F(0,\tau)\right| - \left|F(0,\tau-T)\right|\right),\end{aligned} \qquad (4)$$

where $c = 2gUC^2l/I_0 = \pi gKN$ and $T=\omega_0 C/\Delta t(1+v_0/v_{gr})$ are dimensionless constants of the control circuit.

The numerical solution of equations (1) together with (2) and (4) was carried out by a method of finite differences [1,8]. Figure 3 shows a plot of output amplitude versus time for $l=3.5$. Initially the control is absent, and it is switched in at the moment marked by an arrow. It is well visible that the arisen intensive self-modulation damps after start of the control, and regime of stationary single-frequency generation is reached. Now the additional terms in the ratio (4) vanish, and $A\equiv 1$. It is the same regime as in the stationary theory, but presence of the control makes it stable. The parameters of the control circuit $c=0.95$ and $T=0.8$ were selected empirically to reach the steady generation in the widest range of the dimensionless length l. At these parameters it takes place for $l_{st} < l \le 3.7$. Thus, in comparison with a usual BWO it is possible to lift a threshold of self-modulation in l approximately in 1.27 times, and in the working beam current approximately twice (1.27^3). The dimensionless amplitude of an output signal $|F|$ in this domain is approximately constant. Therefore, efficiency $\eta = 2^{-5/3} I_0^{1/3} U^{-1/3} K^{1/3} |F|^2$ and output power $P = 2^{-5/3} I_0^{4/3} U^{2/3} K^{1/3} |F|^2$, accessible in a regime of single-frequency generation, grow approximately in 1.3 and 2.5 times. Probably, these results may be improved by using more perfect methods of control.

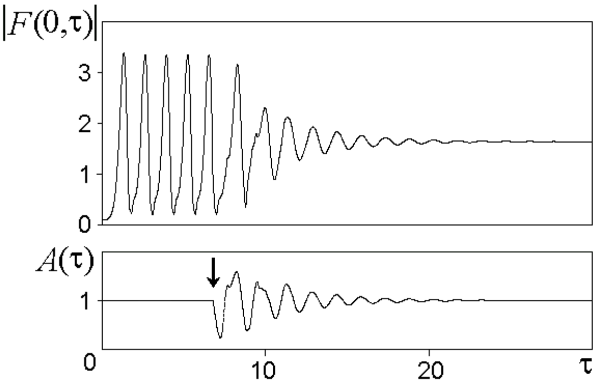

Figure 3: Dimensionless amplitude of output signal in BWO and normalized beam current in dependence on time at l=3.5. An arrow indicates the moment of control switch in. Parameters of the control circuit c=0.95 and T=0.8.

Acknowledgements

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