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MULTIPARAMETRIC CRITICALITY IN A LASER SYSTEM

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Beside chaotic and regular dynamics, the special kind of behaviour occurs in nonlinear systems. This is the critical state arising usually just at the onset of chaos. Its distinctive peculiarity is a presence of the wide band of characteristic temporal scales up to infinitely large ones in the generating signal.

The simplest critical behaviour type was discovered 15 years ago by Feigenbaum [1]. It is associated with road to chaos via period doubling bifurcations under variation of a control parameter. If we undertake a multiparametric consideration of transition to chaos, some other types of criticality may appear in typical situations. Each of these types is characterized by presence of some specific universal hierarchically organized structure in a parameter space having a property of scale invariance.

Here we consider the critical behaviour types taking place in a laser device consisting of two unidirectionally coupled subsystems. The first one is the periodically Q-switched single-mode laser. The second subsystem is also the single-mode laser using four-level scheme pumped by the first laser without some backward influence. The next equations may be obtained for the device dynamics:

$$\dot{\Delta}_1=\gamma_1(r-n_1\Delta_1-\Delta_1),\quad \dot{n}_1=K_1n_1(\Delta_1-1-m\cos\omega t), \tag{1}$$

$$\dot{N}=\Gamma(n_2-N),\quad \dot{\Delta}_2=\gamma_2(kN-n_2\Delta_2-\Delta_2),\quad \dot{n}_2=K_2n_2(\Delta_2-1). \tag{2}$$

Here Δ_1, Δ_2 are population inversions at working levels of the both lasers, n_1, n_2 are the field intensities, N is a population of the highest energy level in the second laser intermediate for inversion obtaining. Parameters K_1 and K_2 characterize the cavity dumping rates, r is the pump rate for the first laser, k is the coupling coefficient, γ_1, γ_2 and Γ are the relaxation rates for corresponding transitions, ω and m are the parameters of cavity dumping modulation for the first laser.

The first subsystem describing by Eqs.(1) is independent of the second one. It is known that it demonstrates transition to chaos via period doubling bifurcations [2]. On the other hand, if we choose the parameters for the first subsystem to demonstrate periodic pulsations, then the period doubling scenario may be expected possible to observe in the second subsystem under variation of its own parameters. We know that the simplest representative of Feigenbaum's universality class is a logistic map [1]. So, we expect that the critical phenomena in the composite system (1,2) will be just the same as in two logistic maps with unidirectional coupling:

$$x_{n+1}=1-\lambda x_n^2,\qquad y_{n+1}=1-Ay_n^2-Bx_n^2\ . \tag{3}$$

Here x and y are the dynamical variables, n is the discrete time, λ and A are the parameters controlling transition to chaos in both subsystems, B is the coupling parameter.

Fig.1 shows the disposition of critical surfaces, lines and points of different type in the three-dimensional parameter space of the model system (3). There are two Feigenbaum critical surfaces $F1$ and $F2$ corresponding to onset of chaos in the first and in the second subsystems, respectively. These surfaces intersect along the bicritical line B. The surface $F2$ is bounded also by the tricritical line T. The lines B and T meet in the multicritical point BT. The second end of the bicritical line is the double Feigenbaum point DF, where the system breaks down two uncoupled critical logistic maps. The universal constants for phase space and parameter space scaling are presented in the Table for all critical situations. (For F and T they were found earlier [1,2], for B and BT we have calculated them numerically by specially developed renormalization group analysis [4].)

Type of criticality	Phase space scaling factors	Parameter space scaling factors
Feigenbaum F	-2.502907876	4.66920161
Tricritical T	-1.690302971	7.28468622 2.85712413
Bicritical B	-2.502907876 -1.505318159	4.66920161 2.39272443
Multicritical BT	-2.502907876 -1.2416604	4.66920161 2.654654 1.54172055
Double Feigenbaum DF	-2.502907876 -2.502907876	4.66920161 4.66920161 2

Also we find all the mentioned critical situations directly in the differential equations (1,2). For example, the bicritical point may be easily found if one changes the parameters of both subsystems (1) and (2) to lead each of them just onto the border of chaos. Fig.2 shows the phase portraits for both lasers in the bicritical point. The attractor has fractal, Cantor-like structure, controlling by scaling factors from the Table.

We believe that investigation of multiparametric criticality has a key significance for understanding of fine hierarchically organized parameter space structure for nonlinear systems demonstrating transition to chaos. Particularly, the presence of somewhat kind of criticality gives a possibility to use such simple models as logistic maps for quantitative description of complicated systems in suitable domains of their parameter space.

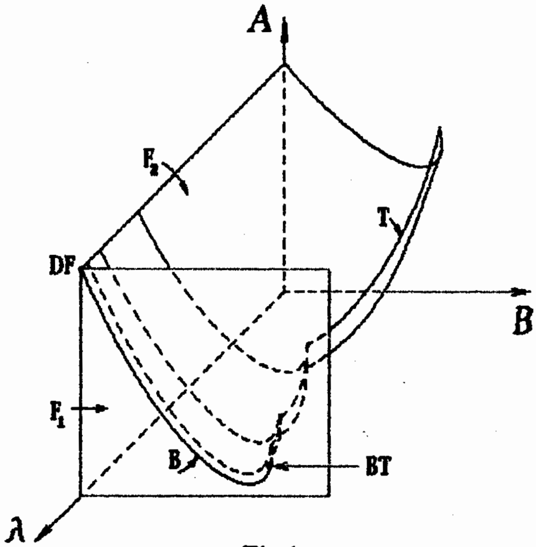

Fig.1

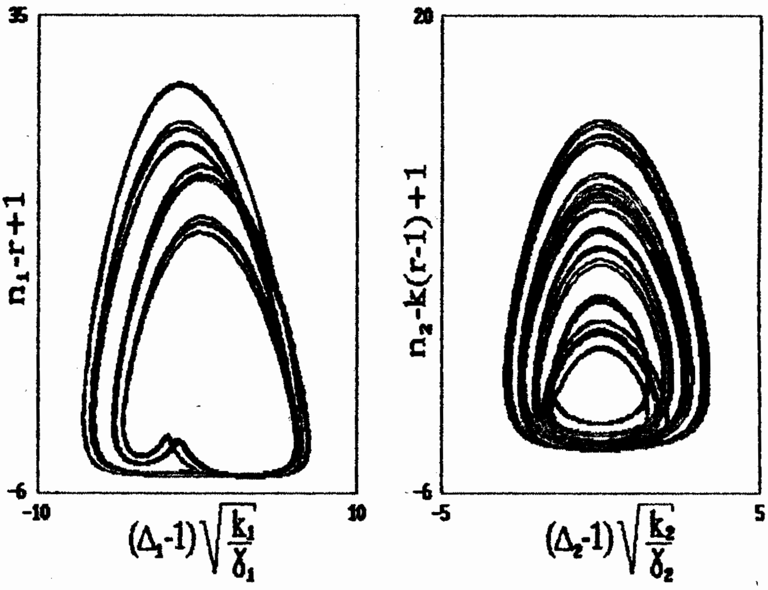

Fig.2

R e f e r e n c e s

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